Revised proof on K-S poles

also denoted as

Denote equivalence class of all measurements $\hat{O}$ constructed ~~by~~ from $\hat{A}, \hat{B}$ by their tensor product as ~~$\{A, B\}$~~ $\{\langle A, B\rangle\}$

Then ~~[illegible]~~

$$\rho_{\hat{O}_1, \hat{O}_2}(X, Y) = 0$$

$$\Rightarrow [O_1 \otimes I]_{\{\langle A,B\rangle\}}(A, B) \neq Y$$

$$\text{or } [I \otimes O_2]_{\{\langle A,B\rangle\}}(A, B) \neq Y.$$

Then we claim

$$[H_S \otimes 1]^{2f}_{\{\langle H_S, H_S\rangle\}}(H_S, H_S) = X$$

$$\Rightarrow [1 \otimes H_S]_{\{\langle H_S, H_S\rangle\}}(H_S, H_S) = X$$

$$\Leftarrow [1 \otimes H_S]_{\{\langle H_S', H_S\rangle\}}(H_S', H_S) = X$$

$$[F(H_S \otimes 1)]_{\{\langle H_S', H_S\rangle\}}(H_S', H_S) = f(X)$$

By OLOC $[1 \otimes H_S]_{\{H_S, H_S\}}(H_S, H_S) = [1 \otimes H_S]_{\{H_S', H_S\}}(H_S, H_S)$

& ELOC $[1 \otimes H_S]_{\{\langle H_S', H_S\rangle\}}(H_S, H_S) = [1 \otimes H_S]_{\{\langle H_S', H_S\rangle\}}(H_S', H_S)$

Here.

$$[f(H_s\,\alpha)]_{\{\langle H_s', H_s\rangle\}}(H_s', H_s)$$

$$= f\left([H_s\,\alpha]_{\{\langle H_s, H_s\rangle\}}(H_s, H_s)\right)$$

but Pure would read

$$\{f(H_s\,\alpha)\}_{\{\langle H_s', H_s\rangle\}}(H_s, H_s)$$

$$= f\left(\{H_s\,\alpha\}\{\langle H_s, H_s\rangle\}(H_s, H_s)\right)$$

Doc says.

$$[\theta, \alpha]_{\{\langle A, B\rangle\}}\overset{D,\,E}{(\cancel{A}, \cancel{B})} = \{\theta, \alpha\}_{\{\langle A, C\rangle\}}\overset{D,\,E}{(\cancel{A}, B)}$$

Bob says.

$$\{\theta, \alpha\}_{\{\langle A, B\rangle\}}\overset{C,\,D}{(\cancel{A}, \cancel{B})} = \{\theta, \alpha\}_{\{\langle A, B\rangle\}}\overset{C,\,E}{(\cancel{A}, \cancel{C})}$$

[illegible] our result is

$$[f(H_s\,\alpha)]_{\{H_s'\,\alpha\}}(H_s'\,\alpha)$$

$$= f\left([H_s\,\alpha]\left(\overset{\cancel{H_s\,\alpha\,H_s}}{H_s\,\alpha}\right)\right)$$

Nonlocality and the Kochen-Specker Paradox

Two charges against realism in QM

1.) algebraic contradiction — Kochen-Specker
2.) nonlocality — Bell

We do two things
1.) show how these two results are linked
2.) ~~Derive~~ give a proof of nonlocality which is nearly algebraic.

Proofs of nonlocality via Bell Inequality have been charged with 'hidden assumptions' as Fine remarked (1974)

'Hidden variables are one thing, hidden assumptions another'.

Fine has claimed (1974) that Bell's original type of proof involved Fine (in the form of the Product Rule (JD) assumption that QM joints and phase-space joints were identical)
Anyway Fine said we are committed to joint distributions for non-commuting observables even in the Stochastic-type proof (1982).
Fine is mistaken here (Bell → Kolm. Joints Kochen Joints → self joint in a model. But this model need not be the real world!)

However all proofs of nonlocality via Bell's theorem do involve notions of

Probability theory that might be challenged.

Query Can we give a purely algebraic proof of nonlocality

Question posed by Bub (1976) in this form

Is it impossible to extend Maczynski's Theorem (1971) to locally maximal observables.

M's theorem says we do not need contextuality to allow value assignments to maximal observables.

If M's theorem cannot be extended this would mean locally maximal observables must be contextual $\Rightarrow$ Nonlocality.

Unfortunately ~~B~~ Demopoulos, Bub & Hardegree have shown that M's theorem can be extended to locally maximal observables.

Two senses of contextual are here being confused.

We distinguish $[Q]^{\phi}_{R}(P)$ from $\{Q\}^{\phi}_{R}(R)$

$$\{Q\}^{\phi}_{R} =_{Df} g(\{R\}^{\phi}) \quad \text{if } Q = g(R)$$

We interpret the number $\{Q\}^{\phi}_{R}$ as the value of the split observable Q_R, i.e. $[Q_R]^{\phi} =_{Df} \{Q\}^{\phi}_{R} =_{Df} g(\{R\}^{\phi})$

i.e. Q_R is measured by measuring R and applying g to the result.

A is ontological contextuality - suffix
ρ is environmental contextuality suffix.
FUNC says $[Q_R] = [Q_{R'}]$ in same environmental context.
Two senses of locality

OLOC. locally measured observables are not holist

ELOC the value possessed by a locally measured observable does not depend on the setting of a remote piece of apparatus.

We show Func* & VR & ELOC & OLOC $\Rightarrow$ contradiction
where VR says $\text{Prob}^{\phi}_{Q}(q) = 0 \Rightarrow [Q] \neq q$.

The steps in the proof are as follows

1.) Derive CVR from VR & FUNC*

If $Q_1 = f(A)$, $Q_2 = g(A)$

$$P_{Q_1, Q_2}(\lambda, \mu) = 0 \Rightarrow [Q_1]_A(A) \neq \lambda \text{ or } [Q_2]_A(A) \neq \mu .$$

N.B. EVR is not derivable from VR and $\Rightarrow$ FUNC

2.) Assuming OLOC for 2 separated systems we obtain

$$P_{Q_1 \otimes 1, 1 \otimes Q_2}(x, y) = 0 \quad \text{where } Q_1 = h(A),\ Q_2 = k(B)$$

$$\Rightarrow [Q_1 \otimes 1]_A(A, B) \neq x$$

$$\text{or } [1 \otimes Q_2]_B(A, B) \neq y$$

Put another way if we can show

$p_{Q\otimes 1, 1\otimes Q_2}(x, y) = 0 \Rightarrow$

then we can conclude.

* $[Q_1\otimes 1]_A(A,B) = x \Rightarrow [1\otimes Q_2]_B(A,B) \neq y$.

2.) We now apply these results to 2 separated spin 1-systems in singlet state of total spin.

We first show

$$\psi = \frac{1}{\sqrt{3}}\left(|a+c\rangle\otimes|a+c\rangle - |b+c\rangle\otimes|b+c\rangle - |a+b\rangle\otimes|a+b\rangle\right)$$

$H_S = aS_x^2 + bS_y^2 + cS_z^2$.

Hence we obtain

$$\begin{cases} p^{\psi}_{H_S\otimes 1, 1\otimes H_S}(x, y) = 0 \quad \forall\, x \neq y. \\ p_{S_z^2\otimes 1, 1\otimes H_S}(f(x'), x) = 0 \quad \forall\, x' \text{ s.t. } f(x') \neq f(x) \end{cases}$$ (a)

where $S_z^2\otimes 1 = f(H_S\otimes 1) = f'(H_S'\otimes 1)$

We now apply * ~~and MWC~~ to derive the following results

$[H_S\otimes 1]^{\psi}(H_S\otimes 1, 1\otimes H_S) = x \Rightarrow [1\otimes H_S]^{\psi}(H_S\otimes 1, 1\otimes H_S) = x$

$[f(H_S\otimes 1)]_{H_S'\otimes 1}(H_S'\otimes 1, 1\otimes H_S) = f(x) \Leftarrow [1\otimes H_S]^{\psi}(H_S'\otimes 1, 1\otimes H_S) = x$

Hence using we

$$\cancel{[H_5 \otimes 7]^{24}(H_5 \otimes 7)}$$

$$\{H_5 \otimes 7\}^{4}(H_5 \otimes 1) = X \Rightarrow \{1 \otimes H_5\}^{4}(1 \otimes H_5) = X$$

$$\Downarrow$$

$$[f(H_5 \otimes 1)]^{4}_{H_5' \otimes 1}(H_5' \otimes 7) = f(X) \Leftarrow [1 \otimes H_5]^{4}(1 \otimes H_5) = X$$

Hence $[f(H_5 \otimes 7)]^{4}_{H_5' \otimes 7}(H_5' \otimes 7)$

$$= f([H_5 \otimes 7]^{4}(H_5 \otimes 7)) \quad **$$

of Fure

$$[f(H_5 \otimes 7]^{4}_{H_5' \otimes I}(H_5 \otimes 7)$$

$$= f(\{H_5 \otimes 7\}^{4}(H_5 \otimes 7))$$

But ** can be used to get a contradiction - since it assigns to 2-dividen a number which is independent of orientation of x, y axes in H_5'.
Repeat argument for x, y directions, then these no's add to 2.

But this is [illegible] $\{1c3$ correct -

violating OLOC means we cannot specify locally maximal observables except by specifying maximal observable O. Solutions relating to the [illegible] system → categorical holism.

Violating ELOC means that we have action-at-a-distance in some sense in the Bell inequality literature.